Data-Driven Filtered Reduced Order Models of Nonlinear Systems

T. Iliescu

Department of Mathematics
ICAM
Virginia Tech

MoRePaS 2018

Nantes
April 11, 2018

National Science Foundation DMS-1522656 2015-2018

Collaborators

- Muhammad Mohebujjaman (postdoc, Virginia Tech)
- Xuping Xie (postdoc, Oak Ridge National Laboratory)
- Leo Rebholz (professor, Clemson University)

- Birgul Koc (student, Virginia Tech)
- Changhong Mou (student, Virginia Tech)

Collaborators

- Muhammad Mohebujjaman (postdoc, Virginia Tech)
- Xuping Xie (postdoc, Oak Ridge National Laboratory)
- Leo Rebholz (professor, Clemson University)
- Birgul Koc (student, Virginia Tech)
- Changhong Mou (student, Virginia Tech)

Collaborators

- Muhammad Mohebujjaman (postdoc, Virginia Tech)
- Xuping Xie (postdoc, Oak Ridge National Laboratory)
- Leo Rebholz (professor, Clemson University)

- Birgul Koc (student, Virginia Tech)
- Changhong Mou (student, Virginia Tech)

VirginiaTech Invent the Future

Collaborators

- Muhammad Mohebujjaman (postdoc, Virginia Tech)
- Xuping Xie (postdoc, Oak Ridge National Laboratory)
- Leo Rebholz (professor, Clemson University)

- Birgul Koc (student, Virginia Tech)
- Changhong Mou (student, Virginia Tech)

Collaborators

- Muhammad Mohebujjaman (postdoc, Virginia Tech)
- Xuping Xie (postdoc, Oak Ridge National Laboratory)
- Leo Rebholz (professor, Clemson University)

- Birgul Koc (student, Virginia Tech)
- Changhong Mou (student, Virginia Tech)

General Problem

- POD
- RBM
- DMD
- systems theoretic ROMs (Serkan ☺)

General Problem

- POD
- RBM
- DMD
- systems theoretic ROMs (Serkan ☺)

General Problem

- POD
- RBM
- DMD
- systems theoretic ROMs (Serkan ☺)

General Problem

- POD
- RBM
- DMD
- systems theoretic ROMs (Serkan ☺)

What is the Problem?

(I) PDE $\dot{\mathbf{u}} = \mathbf{f}(\mathbf{u})$ given

(II) FEM $\left(\dot{\mathbf{u}}_h \, , \mathbf{v}_h\right) = \left(\mathbf{f}(\mathbf{u_h}) \, , \mathbf{v_h}\right)$ given

(III) ROM

- r given
- $\{\varphi_1, \ldots, \varphi_r\}$ given
- model ???

What is the Problem?

(I) PDE $\dot{\mathbf{u}} = \mathbf{f}(\mathbf{u})$ given

(II) FEM $\left(\dot{\mathbf{u}}_h \, , \mathbf{v}_h\right) = \left(\mathbf{f}(\mathbf{u_h}) \, , \mathbf{v_h}\right)$ given

(III) ROM

- r given
- $\{\varphi_1, \ldots, \varphi_r\}$ given
- model ???

What is the Problem?

(I) PDE $\quad \dot{\mathbf{u}} = \mathbf{f}(\mathbf{u})$ given

(II) FEM $\quad \left(\dot{\mathbf{u}}_h \, , \mathbf{v}_h\right) = \left(\mathbf{f}(\mathbf{u_h}) \, , \mathbf{v_h}\right)$ given

(III) ROM

- r given
- $\{\varphi_1, \ldots, \varphi_r\}$ given
- model ???

What is the Problem?

(I) PDE $\quad \dot{\mathbf{u}} = \mathbf{f}(\mathbf{u})$ given

(II) FEM $\quad \left(\dot{\mathbf{u}}_h \, , \mathbf{v}_h\right) = \left(\mathbf{f}(\mathbf{u_h}) \, , \mathbf{v_h}\right)$ given

(III) ROM

- r given
- $\{\varphi_1, \ldots, \varphi_r\}$ given
- model ???

What is the Problem?

(I) PDE $\quad \dot{\mathbf{u}} = \mathbf{f}(\mathbf{u})$ given

(II) FEM $\quad \left(\dot{\mathbf{u}}_h \, , \mathbf{v}_h \right) = \left(\mathbf{f}(\mathbf{u_h}) \, , \mathbf{v_h} \right)$ given

(III) ROM

- r given
- $\{\varphi_1, \ldots, \varphi_r\}$ given
- model ???

VirginiaTech Invent the Future

What is the Problem?

(I) PDE $\quad \dot{\mathbf{u}} = \mathbf{f}(\mathbf{u})$ given

(II) FEM $\quad \left(\dot{\mathbf{u}}_h \, , \mathbf{v}_h \right) = \left(\mathbf{f}(\mathbf{u_h}) \, , \mathbf{v_h} \right)$ given

(III) ROM

- r given
- $\{\varphi_1, \ldots, \varphi_r\}$ given
- model ???

VirginiaTech Invent the Future

Standard Solution — Discretization Levels

(I) PDE $dim = \infty$

(II) FEM $dim = N = \mathcal{O}(10^6)$

(III) ROM $dim = r = \mathcal{O}(10)$

Standard Solution   Discretization Levels

(I) PDE $dim = \infty$

(II) FEM $dim = N = \mathcal{O}(10^6)$

(III) ROM $dim = r = \mathcal{O}(10)$

VirginiaTech Invent the Future

Standard Solution — Discretization Levels

(I) PDE $dim = \infty$

(II) FEM $dim = N = \mathcal{O}(10^6)$

(III) ROM $dim = r = \mathcal{O}(10)$

Standard Solution Model

- $\mathbf{u}_r = \sum_{j=1}^{r} a_j \, \varphi_j$
- $\left(\dot{\mathbf{u}}_r \, , \varphi_i \right) = \left(\mathbf{f}(\mathbf{u_r}) \, , \varphi_\mathbf{i} \right) \qquad \mathbf{i = 1, \ldots, r}$
- Galerkin ROM [1]

[1] NSE $\quad \dot{\boldsymbol{a}} = A \, \boldsymbol{a} + \boldsymbol{a}^\top B \, \boldsymbol{a}$

Virginia Tech Invent the Future

Standard Solution Model

- $\mathbf{u}_r = \sum_{j=1}^{r} a_j \, \varphi_j$

- $\left(\dot{\mathbf{u}}_r \, , \varphi_i \right) = \left(\mathbf{f}(\mathbf{u_r}) \, , \varphi_\mathbf{i} \right) \qquad \mathbf{i} = \mathbf{1}, \ldots, \mathbf{r}$

- Galerkin ROM [1]

[1] NSE $\quad \dot{\boldsymbol{a}} = A \, \boldsymbol{a} + \boldsymbol{a}^\top \, B \, \boldsymbol{a}$

VirginiaTech Invent the Future

Standard Solution Model

- $\mathbf{u}_r = \sum_{j=1}^{r} a_j \, \varphi_j$

- $\left(\dot{\mathbf{u}}_r \, , \varphi_i \right) = \left(\mathbf{f}(\mathbf{u_r}) \, , \varphi_\mathbf{i} \right) \qquad \mathbf{i} = \mathbf{1}, \ldots, \mathbf{r}$

- Galerkin ROM [1]

[1] NSE $\quad \dot{\boldsymbol{a}} = A \, \boldsymbol{a} + \boldsymbol{a}^\top \, B \, \boldsymbol{a}$

VirginiaTech Invent the Future

New Solution Discretization Levels

(I) PDE $dim = \infty$

(II) FEM $dim = N = \mathcal{O}(10^6)$

(III) FEM-ROM $dim = d = \mathcal{O}(10^3)$

(IV) ROM $dim = r = \mathcal{O}(10)$

VirginiaTech Invent the Future

New Solution Discretization Levels

(I) PDE $dim = \infty$

(II) FEM $dim = N = \mathcal{O}(10^6)$

(III) FEM-ROM $dim = d = \mathcal{O}(10^3)$

(IV) ROM $dim = r = \mathcal{O}(10)$

VirginiaTech Invent the Future

New Solution Discretization Levels

(I) PDE $dim = \infty$

(II) FEM $dim = N = \mathcal{O}(10^6)$

(III) FEM-ROM $dim = d = \mathcal{O}(10^3)$

(IV) ROM $dim = r = \mathcal{O}(10)$

New Solution Discretization Levels

(I) PDE $dim = \infty$

(II) FEM $dim = N = \mathcal{O}(10^6)$

(III) FEM-ROM $dim = d = \mathcal{O}(10^3)$

(IV) ROM $dim = r = \mathcal{O}(10)$

New Solution Model

- $\mathbf{u}_d = \sum_{j=1}^{d} a_j \varphi_j$

- $\left(\dot{\mathbf{u}}_d , \varphi_i \right) = \left(\mathbf{f}(\mathbf{u}_d) , \varphi_i \right) \qquad i = 1, \ldots, r$

- $\left(\dot{\mathbf{u}}_r , \varphi_i \right) = \left(\mathbf{f}(\mathbf{u}_r) , \varphi_i \right) + \left[\left(\mathbf{f}(\mathbf{u}_d) , \varphi_i \right) - \left(\mathbf{f}(\mathbf{u}_r) , \varphi_i \right) \right] \qquad i = 1, \ldots, r$

- Galerkin ROM + correction

New Solution Model

- $\mathbf{u}_d = \sum_{j=1}^{d} a_j \varphi_j$

- $\left(\dot{\mathbf{u}}_d , \varphi_i \right) = \left(\mathbf{f}(\mathbf{u_d}) , \varphi_\mathbf{i} \right) \qquad \mathbf{i} = \mathbf{1}, \ldots, \mathbf{r}$

- $\left(\dot{\mathbf{u}}_r , \varphi_i \right) = \left(\mathbf{f}(\mathbf{u_r}) , \varphi_\mathbf{i} \right) + \left[\left(\mathbf{f}(\mathbf{u_d}) , \varphi_\mathbf{i} \right) - \left(\mathbf{f}(\mathbf{u_r}) , \varphi_\mathbf{i} \right) \right] \qquad \mathbf{i} = \mathbf{1}, \ldots, \mathbf{r}$

- Galerkin ROM + correction

VirginiaTech Invent the Future

New Solution Model

- $\mathbf{u}_d = \sum_{j=1}^{d} a_j \varphi_j$

- $\left(\dot{\mathbf{u}}_d \, , \varphi_i \right) = \left(\mathbf{f}(\mathbf{u_d}) \, , \varphi_\mathbf{i} \right) \qquad \mathbf{i} = \mathbf{1}, \ldots, \mathbf{r}$

- $\left(\dot{\mathbf{u}}_r \, , \varphi_i \right) = \left(\mathbf{f}(\mathbf{u_r}) \, , \varphi_\mathbf{i} \right) + \left[\left(\mathbf{f}(\mathbf{u_d}) \, , \varphi_\mathbf{i} \right) - \left(\mathbf{f}(\mathbf{u_r}) \, , \varphi_\mathbf{i} \right) \right] \qquad \mathbf{i} = \mathbf{1}, \ldots, \mathbf{r}$

- Galerkin ROM + correction

VirginiaTech Invent the Future

New Solution Model

- $\mathbf{u}_d = \sum_{j=1}^{d} a_j \varphi_j$

- $\left(\dot{\mathbf{u}}_d , \varphi_i \right) = \left(\mathbf{f}(\mathbf{u_d}) , \varphi_\mathbf{i} \right) \qquad \mathbf{i} = \mathbf{1}, \ldots, \mathbf{r}$

- $\left(\dot{\mathbf{u}}_r , \varphi_i \right) = \left(\mathbf{f}(\mathbf{u_r}) , \varphi_\mathbf{i} \right) + \left[\left(\mathbf{f}(\mathbf{u_d}) , \varphi_\mathbf{i} \right) - \left(\mathbf{f}(\mathbf{u_r}) , \varphi_\mathbf{i} \right) \right] \qquad \mathbf{i} = \mathbf{1}, \ldots, \mathbf{r}$

- Galerkin ROM + correction

VirginiaTech Invent the Future

New Solution Data-Driven Model

- $$\left[\left(\mathbf{f}(\mathbf{u_d}),\varphi_\mathbf{i}\right)-\left(\mathbf{f}(\mathbf{u_r}),\varphi_\mathbf{i}\right)\right]\overset{\text{data-driven}}{\approx}\left(\mathbf{g}(\mathbf{u_r}),\varphi_\mathbf{i}\right)$$

- $$\left(\dot{\mathbf{u}}_r,\varphi_i\right)=\left(\mathbf{f}(\mathbf{u_r}),\varphi_\mathbf{i}\right)+\left(\mathbf{g}(\mathbf{u_r}),\varphi_\mathbf{i}\right)\qquad \mathbf{i}=\mathbf{1},\ldots,\mathbf{r}$$

- Data-Driven Filtered ROM (DDF-ROM) [2]

[2] NSE $\quad \dot{\boldsymbol{a}}=(A+\widetilde{A})\boldsymbol{a}+\boldsymbol{a}^\top(B+\widetilde{B})\boldsymbol{a}$

Virginia Tech Invent the Future

New Solution Data-Driven Model

- $$\left[\left(\mathbf{f}(\mathbf{u_d}) \, , \varphi_\mathbf{i} \right) - \left(\mathbf{f}(\mathbf{u_r}) \, , \varphi_\mathbf{i} \right) \right] \overset{\text{data-driven}}{\approx} \left(\mathbf{g}(\mathbf{u_r}) \, , \varphi_\mathbf{i} \right)$$

- $$\boxed{\left(\overset{\bullet}{\mathbf{u}}_r \, , \varphi_i \right) = \left(\mathbf{f}(\mathbf{u_r}) \, , \varphi_\mathbf{i} \right) + \left(\mathbf{g}(\mathbf{u_r}) \, , \varphi_\mathbf{i} \right) \qquad \mathbf{i} = \mathbf{1}, \ldots, \mathbf{r}}$$

- Data-Driven Filtered ROM (DDF-ROM) [2]

[2] NSE $\overset{\bullet}{\boldsymbol{a}} = (A + \widetilde{A})\boldsymbol{a} + \boldsymbol{a}^\top (B + \widetilde{B})\, \boldsymbol{a}$

VirginiaTech Invent the Future

New Solution   Data-Driven Model

- $$\left[\Big(\mathbf{f}(\mathbf{u_d}) \, , \varphi_\mathbf{i} \Big) - \Big(\mathbf{f}(\mathbf{u_r}) \, , \varphi_\mathbf{i} \Big) \right] \overset{\text{data-driven}}{\approx} \Big(\mathbf{g}(\mathbf{u_r}) \, , \varphi_\mathbf{i} \Big)$$

- $$\boxed{\Big(\dot{\mathbf{u}}_r \, , \varphi_i \Big) = \Big(\mathbf{f}(\mathbf{u_r}) \, , \varphi_\mathbf{i} \Big) + \Big(\mathbf{g}(\mathbf{u_r}) \, , \varphi_\mathbf{i} \Big) \qquad \mathbf{i} = \mathbf{1}, \ldots, \mathbf{r}}$$

- Data-Driven Filtered ROM (DDF-ROM) [2]

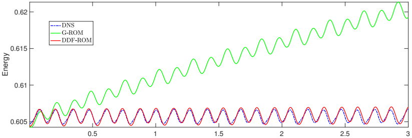
---

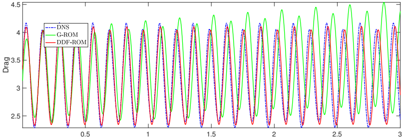
[2]NSE $\quad \dot{\boldsymbol{a}} = (A + \widetilde{A})\boldsymbol{a} + \boldsymbol{a}^\top \, (B + \widetilde{B}) \, \boldsymbol{a}$

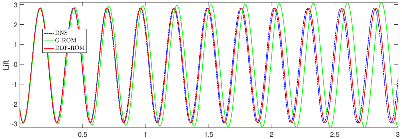
VirginiaTech Invent the Future

DDF-ROM Reproductive

- $Re = 1000$



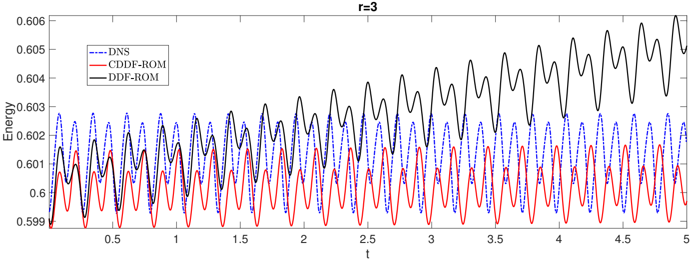




VirginiaTech Invent the Future

DDF-ROM + Constraints Predictive

- $Re = 1000$
- 50% data



VirginiaTech Invent the Future

Correction = Numerical Dissipation

- eddy viscosity ROMs
  - mixing length: Lumley et al
  - Smagorinsky: Noack et al, Ullman & Lang, *Wang et al*, Rozza et al
  - variational multiscale: Iollo et al, *Wang et al*, . . .
  - *dynamic subgrid-scale: Wang et al*

- numerical stabilization
  - Noack et al, Iollo et al, Karniadakis et al, Farhat et al, Amsallem et al, Carlberg et al, Kalashnikova et al, Balajewicz et al, *Wells et al*, . . .
  - Schneier Friday talk
  - Stabile Thursday talk
  - Ali, Hijazi Wednesday poster
  - Taddei Wednesday poster

Correction = Numerical Dissipation

- eddy viscosity ROMs
  - mixing length: Lumley et al
  - Smagorinsky: Noack et al, Ullman & Lang, *Wang et al*, Rozza et al
  - variational multiscale: Iollo et al, *Wang et al*, . . .
  - *dynamic subgrid-scale: Wang et al*
- numerical stabilization
  - Noack et al, Iollo et al, Karniadakis et al, Farhat et al, Amsallem et al, Carlberg et al, Kalashnikova et al, Balajewicz et al, *Wells et al*, . . .
  - Schneier Friday talk
  - Stabile Thursday talk
  - Ali, Hijazi Wednesday poster
  - Taddei Wednesday poster

VirginiaTech Invent the Future

Correction = Numerical Dissipation

- eddy viscosity ROMs
  - mixing length: Lumley et al
  - Smagorinsky: Noack et al, Ullman & Lang, *Wang et al*, Rozza et al
  - variational multiscale: Iollo et al, *Wang et al*, . . .
  - *dynamic subgrid-scale: Wang et al*
- numerical stabilization
  - Noack et al, Iollo et al, Karniadakis et al, Farhat et al, Amsallem et al, Carlberg et al, Kalashnikova et al, Balajewicz et al, *Wells et al*, . . .
  - Schneier Friday talk
  - Stabile Thursday talk
  - Ali, Hijazi Wednesday poster
  - Taddei Wednesday poster

Correction = Numerical Dissipation

- eddy viscosity ROMs
  - mixing length: Lumley et al
  - Smagorinsky: Noack et al, Ullman & Lang, *Wang et al*, Rozza et al
  - variational multiscale: Iollo et al, *Wang et al*, . . .
  - *dynamic subgrid-scale: Wang et al*

- numerical stabilization
  - Noack et al, Iollo et al, Karniadakis et al, Farhat et al, Amsallem et al, Carlberg et al, Kalashnikova et al, Balajewicz et al, *Wells et al*, . . .
  - Schneier Friday talk
  - Stabile Thursday talk
  - Ali, Hijazi Wednesday poster
  - Taddei Wednesday poster

VirginiaTech
Invent the Future

Correction = Numerical Dissipation

- eddy viscosity ROMs
  - mixing length: Lumley et al
  - Smagorinsky: Noack et al, Ullman & Lang, *Wang et al*, Rozza et al
  - variational multiscale: Iollo et al, *Wang et al*, . . .
  - *dynamic subgrid-scale: Wang et al*

- numerical stabilization
  - Noack et al, Iollo et al, Karniadakis et al, Farhat et al, Amsallem et al, Carlberg et al, Kalashnikova et al, Balajewicz et al, *Wells et al*, . . .
  - Schneier Friday talk
  - Stabile Thursday talk
  - Ali, Hijazi Wednesday poster
  - Taddei Wednesday poster

VirginiaTech Invent the Future

Correction = Numerical Dissipation

- eddy viscosity ROMs
  - mixing length: Lumley et al
  - Smagorinsky: Noack et al, Ullman & Lang, *Wang et al*, Rozza et al
  - variational multiscale: Iollo et al, *Wang et al*, . . .
  - *dynamic subgrid-scale: Wang et al*
- numerical stabilization
  - Noack et al, Iollo et al, Karniadakis et al, Farhat et al, Amsallem et al, Carlberg et al, Kalashnikova et al, Balajewicz et al, *Wells et al*,. . .
  - Schneier Friday talk
  - Stabile Thursday talk
  - Ali, Hijazi Wednesday poster
  - Taddei Wednesday poster

VirginiaTech Invent the Future

Correction = Numerical Dissipation

- eddy viscosity ROMs
  - mixing length: Lumley et al
  - Smagorinsky: Noack et al, Ullman & Lang, *Wang et al*, Rozza et al
  - variational multiscale: Iollo et al, *Wang et al*, . . .
  - *dynamic subgrid-scale: Wang et al*
- numerical stabilization
  - Noack et al, Iollo et al, Karniadakis et al, Farhat et al, Amsallem et al, Carlberg et al, Kalashnikova et al, Balajewicz et al, *Wells et al*,. . .
  - Schneier Friday talk
  - Stabile Thursday talk
  - Ali, Hijazi Wednesday poster
  - Taddei Wednesday poster

Correction = Numerical Dissipation

- eddy viscosity ROMs
  - mixing length: Lumley et al
  - Smagorinsky: Noack et al, Ullman & Lang, *Wang et al*, Rozza et al
  - variational multiscale: Iollo et al, *Wang et al*, . . .
  - *dynamic subgrid-scale: Wang et al*
- numerical stabilization
  - Noack et al, Iollo et al, Karniadakis et al, Farhat et al, Amsallem et al, Carlberg et al, Kalashnikova et al, Balajewicz et al, *Wells et al*,. . .
  - Schneier Friday talk
  - Stabile Thursday talk
  - Ali, Hijazi Wednesday poster
  - Taddei Wednesday poster

VirginiaTech
Invent the Future

Correction = Numerical Dissipation

- eddy viscosity ROMs
  - mixing length: Lumley et al
  - Smagorinsky: Noack et al, Ullman & Lang, *Wang et al*, Rozza et al
  - variational multiscale: Iollo et al, *Wang et al*, ...
  - *dynamic subgrid-scale: Wang et al*
- numerical stabilization
  - Noack et al, Iollo et al, Karniadakis et al, Farhat et al, Amsallem et al, Carlberg et al, Kalashnikova et al, Balajewicz et al, *Wells et al*,...
  - Schneier Friday talk
  - Stabile Thursday talk
  - Ali, Hijazi Wednesday poster
  - Taddei Wednesday poster

VirginiaTech *Invent the Future*

Correction = Numerical Dissipation

- eddy viscosity ROMs
  - mixing length: Lumley et al
  - Smagorinsky: Noack et al, Ullman & Lang, *Wang et al*, Rozza et al
  - variational multiscale: Iollo et al, *Wang et al*, . . .
  - *dynamic subgrid-scale: Wang et al*
- numerical stabilization
  - Noack et al, Iollo et al, Karniadakis et al, Farhat et al, Amsallem et al, Carlberg et al, Kalashnikova et al, Balajewicz et al, *Wells et al*,. . .
  - Schneier Friday talk
  - Stabile Thursday talk
  - Ali, Hijazi Wednesday poster
  - Taddei Wednesday poster

VirginiaTech
Invent the Future

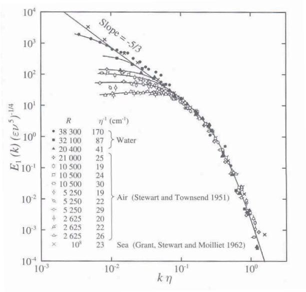
Correction = Numerical Dissipation

- eddy viscosity ROMs
  - mixing length: Lumley et al
  - Smagorinsky: Noack et al, Ullman & Lang, *Wang et al*, Rozza et al
  - variational multiscale: Iollo et al, *Wang et al*, ...
  - *dynamic subgrid-scale: Wang et al*
- numerical stabilization
  - Noack et al, Iollo et al, Karniadakis et al, Farhat et al, Amsallem et al, Carlberg et al, Kalashnikova et al, Balajewicz et al, *Wells et al*,...
  - Schneier Friday talk
  - Stabile Thursday talk
  - Ali, Hijazi Wednesday poster
  - Taddei Wednesday poster

VirginiaTech
Invent the Future

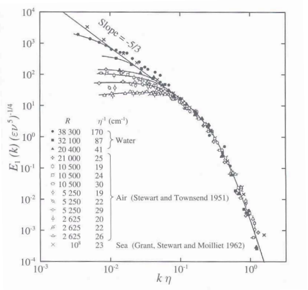
Galerkin ROM Stable, Not Accurate

- stable
  - $\|\mathbf{u}_r^n\|^2 + \nu \, \Delta t \sum_{n=1}^{M} \|\nabla \mathbf{u}_r^n\|^2 \leq \text{data}$
- not accurate



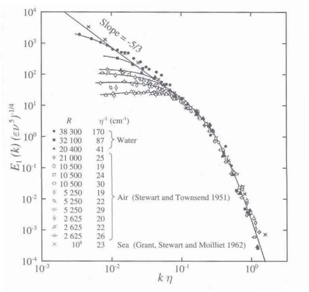
Galerkin ROM Stable, Not Accurate

- stable
  - $\|\mathbf{u}_r^n\|^2 + \nu \, \Delta t \sum_{n=1}^{M} \|\nabla \mathbf{u}_r^n\|^2 \leq \text{data}$
- not accurate



Galerkin ROM Stable, Not Accurate

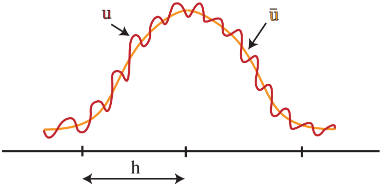
- stable
  - $\|\mathbf{u}_r^n\|^2 + \nu\,\Delta t \sum_{n=1}^{M} \|\nabla \mathbf{u}_r^n\|^2 \leq \text{data}$
- not accurate



VirginiaTech
Invent the Future

Connections

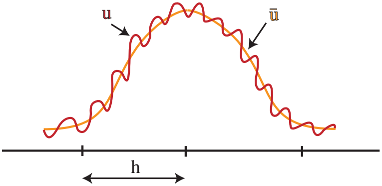
- Large Eddy Simulation



- Variational Multiscale
- Nonlinear Galerkin

Connections

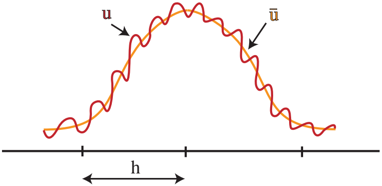
- Large Eddy Simulation



- Variational Multiscale
- Nonlinear Galerkin

Connections

- Large Eddy Simulation



- Variational Multiscale
- Nonlinear Galerkin

Related Work

- no ROM
  - 20th century Adrian, Moser
  - 21st century Iaccarino, Duraisamy, Ling, Xiao
- ROM
  - 21st century Chorin, Duraisamy, Codina, Majda, San, Liu

Related Work

- no ROM
  - 20th century Adrian, Moser
  - 21st century Iaccarino, Duraisamy, Ling, Xiao
- ROM
  - 21st century Chorin, Duraisamy, Codina, Majda, San, Liu

Related Work

- no ROM
  - 20th century Adrian, Moser
  - 21st century Iaccarino, Duraisamy, Ling, Xiao
- ROM
  - 21st century Chorin, Duraisamy, Codina, Majda, San, Liu

Related Work

- no ROM
  - 20th century Adrian, Moser
  - 21st century Iaccarino, Duraisamy, Ling, Xiao
- ROM
  - 21st century Chorin, Duraisamy, Codina, Majda, San, Liu

Related Work

- no ROM
  - 20th century — Adrian, Moser
  - 21st century — Iaccarino, Duraisamy, Ling, Xiao
- ROM
  - 21st century — Chorin, Duraisamy, Codina, Majda, San, Liu

Conclusions

- Galerkin ROM + correction
- correction data-driven
- $Re = 100$, $Re = 500$, $Re = 1000$
  - reproductive ✓
  - predictive (cross-validation) ✓

Conclusions

- Galerkin ROM + correction
- correction data-driven
- $Re = 100$, $Re = 500$, $Re = 1000$
  - reproductive ✓
  - predictive (cross-validation) ✓

Conclusions

- Galerkin ROM + correction
- correction data-driven
- $Re = 100$, $Re = 500$, $Re = 1000$
  - reproductive ✓
  - predictive (cross-validation) ✓

Conclusions

- Galerkin ROM + correction
- correction data-driven
- $Re = 100$, $Re = 500$, $Re = 1000$
  - reproductive ✓
  - predictive (cross-validation) ✓

Conclusions

- Galerkin ROM + correction
- correction data-driven
- $Re = 100$, $Re = 500$, $Re = 1000$
  - reproductive ✓
  - predictive (cross-validation) ✓